

The University of Chicago

A STUDY OF THE DIFFRACTION GRATING AT GRAZING INCIDENCE

PART OF A DISSERTATION SUBMITTED TO
THE FACULTY OF THE DIVISION OF THE
PHYSICAL SCIENCES IN CANDIDACY FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS

1936

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LUIS WALTER ALVAREZ

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The Diffraction Grating at Grazing Incidence

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(Received March 5, 1936)

An experimental test of the grating law has been made by using a coarse, plane grating at grazing incidence, in a spectrograph similar to that used in the x-ray case. No deviations from the grating law were found which were as large as the experimental error of 0.02 percent.

INTRODUCTION

ALTHOUGH the relative values of x-ray wave-lengths are known with considerable accuracy, there has been a good deal of debate upon the subject of their absolute magnitudes. The tabulated wave-lengths are calculated from crystal diffraction experiments, on the assumption that the crystals are perfect, and that the electronic charge has the value 4.77×10^{-10} e.s.u. Bearden¹ has shown that the wave-lengths, as determined from ruled grating x-ray spectra, are uniformly 0.2 percent higher than the crystal values. Since the grating space of the crystal depends upon $e^{\frac{1}{2}}$, it was difficult to believe that the electronic charge could be in error by 0.6 percent, the amount needed to bring the wave-lengths into coincidence. The suggestion that the crystal planes near the surface might have a greater spacing than those in the interior was investigated experimentally, with negative results, and the possibility that there might be some unsuspected error in the grating formula when applied to the case of grazing incidence was considered theoretically by Eckart,² who showed that the usual equations should be valid in this case.

It must be remembered, however, that the grating law has not been thoroughly investigated from the experimental standpoint. Most present day optical wave-lengths are measured from grating spectra, but the grating formula is not used. The wave-lengths are merely determined by comparison with known standards, which have in turn been measured by interferometric means. The last precision measurement of optical wave-lengths in terms of grating space and diffraction angles was made in 1888 by Bell,³ who

used four of Rowland's best plane gratings. It is interesting to note that the value he published for the wave-length of the D_1 line of sodium is higher than the interferometer value by ten times the probable error of his experiments. Since this discrepancy is in the same direction as that of the x-ray case, and since he used angles near 45° , it seemed worth while to investigate the possibility that there might be some systematic error in grating measurements which would become more noticeable as the angle of incidence was increased. (While the work has been in progress, however, several papers have appeared which would make this latter view difficult to entertain.)^{4, 5}

METHOD

As an experimental test of the possibility mentioned above, it was decided to measure the wave-length of a line in the optical region by means of the technique perfected by Bearden.¹ In order that the suspected systematic errors should be the same in this work with optical radiation as in Bearden's work with x-radiation, one might suppose that all linear dimensions of the apparatus ought to be increased in the same ratio as the wave-lengths. Such an increase was prohibited by practical considerations, but the various parts of the apparatus designed by Bearden were duplicated on an enlarged scale, even though the factor was necessarily different for different parts. A diagram of the apparatus is shown in Fig. 1. The wave-length and grating space were increased by a factor of 2400, the slits by 100, and the source to plate distance by about 10. The source of light ($\lambda = 4358\text{\AA}$) was a water cooled mercury arc operating at atmospheric pressure. Each part of the long spectro-

¹ J. A. Bearden, *Phys. Rev.* **37**, 1210 (1931).

² C. Eckart, *Phys. Rev.* **44**, 12 (1933).

³ L. Bell, *Phil. Mag.* **25**, 350 (1888).

⁴ M. Söderman, *Nature* **135**, 67 (1935).

⁵ G. Kellström, *Nature* **136**, 682 (1935).

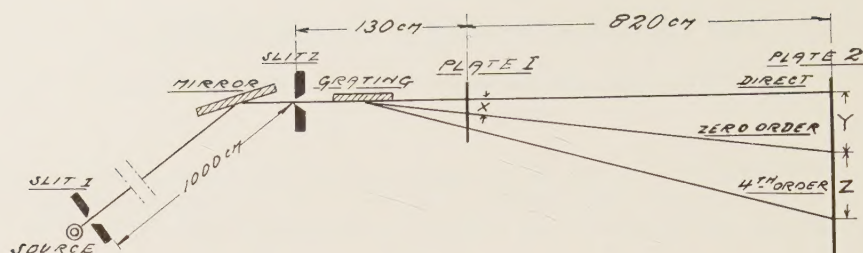


FIG. 1.

graph was mounted on a pillar of granite blocks. The grating was ruled on a seasoned surface plate casting, which was later polished flat to within less than one wave-length. The grooves were milled accurately, by setting with a microscope on a standard scale carried on the milling machine carriage. The width of the grooves was one-half the grating space.

The various pieces of apparatus were aligned on the same horizontal plane to within 0.1 cm with the aid of a surveyor's level, and adjusted vertically to within about 20 seconds of arc, by a modified Gauss eyepiece technique. It was most important to have the second slit accurately parallel to the grating surface, and this was accomplished by rotating the grating until the Lloyd type fringes had their maximum intensity. These Lloyd fringes were caused by scattered light interfering with the relatively weak light diffracted into the higher orders. They introduced an asymmetry into the diffracted line shapes, but were eliminated in the final sets of plates by a shield which screened off the scattered light.

MEASUREMENTS

The distance between the plateholders was measured with a surveyor's tape, which was later calibrated under the same tension against a standard meter. The grating space was compared directly with a standard decimeter scale, using two microscopes. The lines on the second plates were from 5 to 10 mm wide, so their centers could not be located without the use of a microphotometer. A fiducial mark was placed near the center of each line, and the distances between these marks measured on a comparator. Then each line was microphotometered, and its center of gravity located. By knowing the magnification ratio of the instrument, it was possible to deter-

mine the small distance between the fiducial mark and the center of the line. This value was then used as a correction to the comparator measured distance, to give the actual distance between the lines.

A fiducial line was ruled on either side of the diffracted line, with a razor. The reference marks for the direct and reflected beams were impressed on the plate during the exposure, and were the shadows of fine wires stretched vertically across the plateholder in such a way as to bisect the broad lines. The center of a line was found directly from its microphotometer trace. The lines were quite symmetrical, so it was merely necessary to mark the mean abscissa on the trace at several different ordinates; the points lay on a vertical line defining the center of the wide line. The midpoint of the fiducial mark, or pair of marks, was found in the same manner.

EXPERIMENTAL PROCEDURE

Before each series of plates was exposed, the adjustment of the slits, mirror, and grating was rechecked. The method of taking exposures was similar to that used by Bearden. The grating was moved but once, so that no errors could occur from resetting. "Creeping" of the grating carriage along its ways was eliminated by allowing it to stand fifteen minutes after it had been moved. Errors were introduced into the first set of measurements by thermal "boiling" of the images on the plate. This trouble was avoided by lengthening the exposures and guarding against air currents in the room. The shape of the diffracted lines was found to vary greatly with changes in the width of the second slit, angle of diffraction, and order. Under some conditions, the lines appeared as widely spaced doublets or triplets, but they could be focused by adjusting

the width of the grating slit. A small effect of this kind has been noted with optical gratings before.⁶ The line shapes could be predicted accurately by considering the images of the two slits in the grating, as seen in the n th order of diffraction. In the zeroth order, of course, these image slits were the same distance from the grating as the actual slits, but in higher orders, their distances varied with the angles and the order. The line shapes were the calculated Fresnel diffraction patterns from a single slit, and could be obtained with the aid of a Cornu spiral. In the final set of plates, the slit width was adjusted to give the sharpest possible shape of the 4th-order blue line.

CALCULATIONS

The usual grating law $n\lambda = d(\sin i + \sin i')$ may be written more conveniently for the purpose of the experiment in the form:

$$n\lambda = 2d \sin [(2\theta + \alpha)/2] \sin (\alpha/2),$$

where θ is the glancing angle of incidence, and α the angle between the reflected and diffracted beams. Stauss and Porter have shown that there is a third order correction to this formula, which takes into account the horizontal divergence of the beam collimated by the slits. The complete equation becomes:

$$n\lambda = 2d \{ \sin [(2\theta + \alpha)/2] \sin (\alpha/2) \times \{ 1 + [3a^2/20(\phi^2 - \theta^2)](\phi^2/l_2^2 - \theta^2/l_1^2) \},$$

where a is the length of the illuminated portion of the grating, l_1 the distance between the grating and the source, and l_2 the distance between grating and image.

Calculation shows that this correction is negligible for the case of the second plateholder, but that it is appreciable for the first. The only diffracted line recorded on the first plate is the zero order, and for this the Stauss-Porter correction reduces to the form:

$$x = x' \{ 1 + [3a^2/40](1/l_1^2 - 1/l_2^2) \},$$

where x is the measured distance between the direct and reflected lines, and x' is the value to be used in its place in the calculations. This factor affects the calculated wave-length by 0.01

TABLE I. Sample calculation, Plate 90, correction terms from microphotometer traces in cm.

	TRACE DISTANCE	MEAN T.D.	PLATE DISTANCE
Direct Beam	-0.97, -0.94,	-0.96	-0.0192
Zero Order	-1.20, -1.32,	-1.23	-0.0246
4th Order Blue	-1.17 +0.29	+0.29	+0.0058
	COMPARATOR SETTING	CORRECTION	CORRECTED SETTING
Direct Beam	0.0505	-0.0192	0.0313
Zero Order	20.9150	-0.0246	20.8904
4th Order	39.5269		
4th Order	39.7495		
4th Order, Mean	39.6382	+0.0058	39.6440

cm	cm	
20.8904	2.2559	
0.0313	2.2570	
	2.2572	$\theta = 0^\circ 38' 52.39''$
$y = 20.8591$	$x = 2.2567$	$\alpha = 1^\circ 9' 49.99''$
39.6440		$d = 4.0000$ cm
20.8904		$b = 822.30$ cm
		$\lambda = 4358.82 \text{ \AA}$
$z = 18.7536$		

or 0.02 percent, depending upon the position of the first plateholder.

The angles θ and α were calculated from the measured distances x , y , z , and b , by the use of the formulae:

$$\theta = \frac{1}{2} \sin^{-1} [y/(y^2 + (yb/y - x)^2)^{\frac{1}{2}}]$$

$$\alpha = \sin^{-1} [z \cos 2\theta / \{ (y+z)^2 + (yb/y - x)^2 \}^{\frac{1}{2}}].$$

Andoyer's 15 place trigonometric tables were used throughout, and all computations were made on a machine (see Table I).

RESULTS

For the final measurements, two sets of five plates were exposed. In the first set, the grating slit was 1.06 mm wide, and the first holder was about 50 cm from the grating. In the second set, the holder was moved to a point one meter from the grating, and the slit width was increased to 1.38 mm. The lines on the plates of the second group were sharper, since there was less diffraction from the slit, and the correction term was not so sensitive to errors in the determination of the length of illumination on the grating. This length was calculated from the geometry of the apparatus, and also measured on a plate which had been exposed while fastened to the surface of the grating.

The values for the wave-length of the blue line were:

⁶ D. C. Stockbarger and L. Burns, J. O. S. A. **23**, 379 (1933).

4362.93	4359.84
59.86	57.00
60.28	57.49
57.43	57.68
58.43	58.78

Because of the differences in the experimental arrangement, the second five plates were given twice as much weight as the first. The weighted mean of the ten determinations is then $4358.71 \pm 0.35\text{\AA}$. This probable error is from considerations of internal consistency alone. To calculate the actual probable error, we must include the errors in the measurement of b and d . b was known to better than 0.1 cm, and as it enters as a square into the calculation, the limiting error in the wave-length from this source is 0.02 percent. The graph of grating space showed a variation from one part of the surface to another; the spacing near the center was smaller than that near the edges. It was possible to locate the portion of the grating used in the

taking of a particular plate, and the grating space at this section was then employed in the calculation of the wave-length from that plate. This could introduce an error into the determination, of the order of 0.01 percent. We may then conclude that the wave-length of the blue mercury line, as measured in this way is $4358.71 \pm 0.8\text{\AA}$. The accepted value, as given in Kayser's tables, is $4358.30\text{\AA}$. This agreement upholds the view that the diffraction law is valid at grazing incidence. Further evidence that the grating law is accurate at glancing angles may be inferred from the fact that associated predictable phenomena, such as the Stauss-Porter correction and the variation of the line shapes, have been verified.

In conclusion, the author wishes to thank Professors H. G. Gale, A. H. Compton, and Carl Eckart for many helpful discussions of various aspects of the problem.



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